Minimum Time Maneuvering of a Ship, with Wind Disturbances

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Minimum Time Maneuvering of a Ship, with Wind Disturbances

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Abstract. A ship's minimum-time maneuvering problems in the face of wind disturbances are formulated here as a nonlinear, two-point boundary-value problem in the calculus of variations, where is solved using the conjugate gradient restoration method proposed by Miele et al.

Key Words. Ship's Minimum-time Maneuvering , Two-Point Boundary-Value Problem, Conjugate Gradient Restoration Method.

1. INTRODUCTION

It is very important for a ship's master to draw up a ship-handling plan before approaching a berth, leaving it, altering the heading and so on. One possible way of finding a satisfactory plan in advance is to use a ship-handling simulator, and to choose the best method after various trials. However, there are individual differences among the ways chosen by those trials. Unlike this, a mathematical method, using some optimal theory, would be more reliable, if the mathematical model representing a ship's maneuvering motion was accurate. However, it must be noted that the model becomes highly nonlinear, especially at low speeds and for large maneuvering motions such as these used in berthing.

In order to take enough account of the nonlinearity, the authors have formulated these problems as a nonlinear, two-point boundary-value problem in the calculus of variations. This problem has been solved, using the numerical method developed by Miele and his associates over the past few years, called the conjugate gradient restoration (CGR) method (Wu and Miele, 1980; Miele and Iyer, 1970).

Unfortunately, though the solution does not yield on-line control laws, it can be considered that the information or diagrams gained thereby are useful for drawing up a maneuvering plan before the actual ship-handling is taken place. The problems which have already been solved by using this method, can be listed as follows (Shoji, 1992; Ohtsu and Shoji, 1994):

1. The minimum-time course-alteration problem,
2. The minimum-time stopping problem,
3. The minimum-time parallel deviation problem .

However, all of these have previously been solved under conditions with no disturbances.

The problems treated in this paper are two typical patterns in ship-handling with wind disturbances. The ship chosen as the object of the study is *T.S.Shioji Maru* (425 gross tonnage), which is equipped with a bow and a stern thruster, besides a rudder and a controllable-pitch propeller(CPP).

2. FORMULATION OF THE MINIMUM-TIME MANEUVERING PROBLEM

2.1. *Minimum-time Maneuvering Problem*

Let the minimum-time maneuvering problem treated here be defined as follows:

Assume that a ship is travelling at a certain speed in a given direction, at an initial approach point. A ship's master must make her alter course to reach a destination point. Her bearing and speed at the destination point are either free or given. How should he steer her, and use her engine and thrusters, in order to accomplish the work in the minimum time?

2.2. *The Formulation as a Two-point Boundary-value Problem*

The problem stated above might be formulated as two-point boundary-value problem in the calculus of variations as follows:

Let x be defined as the $n(=4)$-dimensional state vector, whose elements are composed of the forward speed u, the sideways speed v and the rate of turn r. u is the $m(=2 or 4)$-dimensional control vector, whose elements are rudder angle δ, CPP blade angle θ_P and power of the bow and stern thrusters, T_b and T_s, respectively. Furthermore, it is assumed that the independent variable is represented by the actual time θ but a time normalization is used to simplify the computations. Thus, θ is replaced by the normalized time $t = \theta/\tau$, which is defined in such a way that the initial time is t=0 and the final time is t=1. Since τ is free in the minimum-time maneuvering problem, this is regarded as the parameter to be optimized.

Using the above notation, this type of minimum-time maneuvering problem can be formulated as follows (Shoji and Ohtsu, 1992):

Minimize the functional

$$I \doteq \int_0^1 f(x, u, \tau, t)dt = \int_0^1 \tau dt = \tau \qquad (1)$$

with respect to the state x, the control u and the τ which satisfy:

1) the differential constraints,

$$\dot{x} - \phi(x, u, \tau, t) = o, \qquad 0 \le t \le 1 \qquad (2)$$

where ϕ denotes a nonlinear hydrodynamic model for representing ship's motions, and

2) the boundary conditions:

i) The initial ship's state,

$$x(o) = given. \qquad (3)$$

ii) The final state of the ship, specified by the function

$$[\psi(x, \tau)]_1 = o, \qquad (4)$$

where the function ψ is a q dimensional vector $(0 \le q \le n)$.

In order to increase reality, the non-differential constraints:

$$S(u, \tau, t) = o, \qquad 0 \le t \le 1, \qquad (5)$$

may be added, by which it is possible to set the maximum limits of rudder angle, propeller blade angle, and power of the bow and stern thrusters, to be applied.

Remarks: In many cases, the constraints on the control variables are given by inequality equations. For example, since the rudder angle, δ, must be restricted to the hard-over angle of δ_{max},

$$-\delta_{max} \le \delta \le \delta_{max} \qquad (6)$$

must hold. In order to obtain the equality given by eq.(5), introducing a new independent variable of δ_d, eq.(6) is transformed to

$$\delta = \delta_{max} \sin \delta_d. \qquad (7)$$

3. MATHEMATICAL MANEUVERING MODEL

3.1. *Basic Equation of Motion*

Table 1 shows *T.S.Shioji Maru*'s principal dimensions.

Table 1 Principal Dimensions of *T.S.Shioji Maru*

Length	49.93 m
Breadth	10.00 m
Tonnage	425.0 GT
Propeller	CPP
Bow Thruster	2.4 tons
Stern Thruster	1.8 tons

This ship is equipped with bow and stern thrusters for low-speed maneuvering, besides a single rudder and a single propeller. The propeller revolution are regulated by a change of the propeller pitch angle.

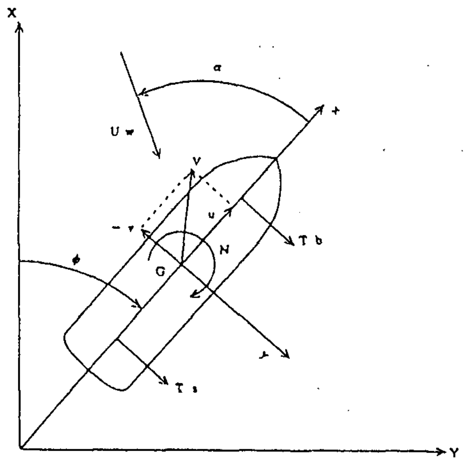

Fig.1 Coordinate System

Figure 1 depicts the ship's fixed-coordinate system. Referencing to this, the mathematical model is written by

$$\begin{aligned} (m + m_x)\dot{u} - mvr &= X_H + X_R + X_P + X_W \\ (m + m_y)\dot{v} + mur &= Y_H + Y_R + Y_T + Y_W \end{aligned} \qquad (8)$$

$$(I_{zz} + J_{zz})\dot{r} = N_H + N_R + N_T + N_W,$$

where m and I_{zz} are the mass and the turning moment of inertia. m_x, m_y and J_{zz} are the added masses along the x and y axes and the added moment of inertia around the z axis. u, v and r are the ship's speed along the x and y axes, and the rate of turn around the z axis, respectively. The subscripts H, P, R, T and W denote the hydrodynamic forces induced by the hull, propeller, rudder, thrusters and wind disturbances, respectively.

3.2. *Hydrodynamic Forces*

The concrete hydrodynamic forces are written by polynomial representations as follows:

$$\begin{aligned} X_H &= -C_t u|u| + X_v Vv + X_{vr} vr \\ &+ X_r Vr + X_{vv} v^2 + X_{rr} r^2 \\ Y_H &= Y_v Vv + Y_{vv} v|v| + Y_r Vr \\ &+ Y_{rr} r|r| + Y_{vr} v|r| \\ N_H &= N_v Vv + N_{vv} v|v| + N_r Vr \\ &+ N_{rr} r|r| + N_{vr} v|r| \end{aligned}$$

where X_v, for example, means $\partial X/\partial v$, etc., and V, the ship's ordinary speed.

The thrust force of the propeller at pitch angles θ_p are as follows:

$$\begin{aligned} X_P &= (1-t)\rho n^2 D_P^4 (C_0 + C_1\theta_P + C_2 J_P \\ &+ C_3\theta_P J_P + C_4\theta_P^2 \\ &+ C_5 J_P^2 + C_6\theta_P^2 J_P + C_7\theta_P J_P^2 \\ &+ C_8\theta_P^3 + C_9 J_P^3) \end{aligned} \tag{9}$$

where t and w_P denote the thrust deduction fraction and the wake fraction. n, D_P and J_p are the propeller revolutions, its diameter and the advance coefficient. $C_1 \sim C_9$ are various empirical coefficients.

The rudder forces at a rudder angle δ are represented by

$$\begin{aligned} X_R &= -(1-t_R)F_N \sin\delta \\ Y_R &= -(1+a_H)F_N \cos\delta \\ N_R &= -(x_R + a_H x_H)F_N \cos\delta, \end{aligned}$$

where t_R,a_H and x_H denote empirical coefficients due to hull-propeller interactions. x_R is the rudder position. The rudder normal force F_N is simplified by

$$F_N = \frac{1}{2}\rho A_R f_\alpha (U_R^2 \sin\delta + \gamma_R(v + l_R r)U_R \cos\delta)$$

where A_R and f_α denote the projected rudder area and its normal force coefficient. γ_R,l_R are empirical coefficients representing the fairing effects of the stream behind the hull. The effective rudder inflow,

$$U_R = (\epsilon - k_x)(1 - w_P)u + k_x(0.7\pi D_P n)tan\theta_P$$

where ϵ denotes the ratio of axial velocity at propeller position to rudder position. k_x denotes the propeller acceleration fraction, simplified by

$$k_x = k_{x_0}\left(\frac{1}{1+e^{-v\theta_P}}\right),$$

using a sigmoid function to represent the discontinuity of k_x, where k_{x_0} means k_x at $\theta_P > 0$. The last two variables, U_R and k_x, were simplified in order to facilitate partial differentiation by the state or control variables. The actuator's dynamics are also considered in the model. For details of the model, see (Shoji and Ohtsu, 1992).

3.3. *Wind Disturbances*

Wind loads on the ship's superstructure can be represented as follows:

$$X_W = \frac{1}{2}\rho_a A_{of} U_W^2 C_X \tag{10}$$

$$Y_W = \frac{1}{2}\rho_a A_{os} U_W^2 C_Y \tag{11}$$

$$N_W = \frac{1}{2}\rho_a A_{os} L_{pp} U_W^2 C_N \tag{12}$$

where C_X, C_Y and C_N denote the experimental coefficients of wind force and moment acting on the ship's superstructure. ρ_a is the density of air. Aof and Aos are the lateral and transverse projected areas of the superstructure, respectively. Since the coefficients C_X,C_Y and C_N are functions of the wind direction relative to the ship, they can be approximated by (Fossen, 1994):

$$C_X = \hat{C}_X \cos\alpha \tag{13}$$

$$C_Y = \hat{C}_Y \sin\alpha \tag{14}$$

$$C_N = \hat{C}_N \sin 2\alpha. \tag{15}$$

4. OPTIMIZATION TECHNIQUE

4.1. *Optimal Conditions*

The above problem can be solved using the theory of calculus of variations. This is referred to as an example of the Bolza type of problem, and it can be recast as a problem of minimizing the augmented functional

$$\begin{aligned} J &= \int_0^1 (f + \lambda^T(\dot{x} - \phi) + \rho^T S)dt \\ &+ (\mu^T \psi)_1 \end{aligned}$$

$$= \int_0^1 (f - \lambda^T \phi + \rho^T S - \dot{\lambda}^T x)dt$$
$$+ \ (\lambda^T x + \mu^T \psi)_1 - (\lambda^T x)_0 \qquad (16)$$

subject to equations (2) – (5), where λ, ρ are variable Lagrange multipliers and μ is a constant Lagrange multiplier. The second equation arises after the customary integration by parts is performed. The functions $x(t)$, $u(t)$ and τ and the multipliers $\lambda(t)$, $\rho(t)$ and μ must satisfy equations (2) – (5) and the following optimality conditions

$$f_u - \phi_u \lambda + S_u \rho = 0, 0 \le t \le 1 \qquad (17)$$
$$\dot{\lambda} - f_x + \phi_x \lambda = 0, 0 \le t \le 1 \qquad (18)$$
$$\int_0^1 (f_\tau - \phi_\tau \lambda + S_\tau \rho)dt + (\psi_\tau \mu)_1 = 0 \qquad (19)$$
$$(\lambda + \psi_x \mu)_1 = 0 \qquad (20)$$

4.2. *Sequential Gradient Restoration Method*

Since the differential systems (2)-(5) have nonlinear properties, it is impossible to find an analytical solution. Thus, approximate and iterative numerical methods are employed to find it. The numerical method used in this paper is the conjugate gradient-restoration method developed by Miele et al.(Wu and Miele, 1980; Miele and Iyer, 1970). In this method, the constraint error,

$$P = \int_0^1 N(\dot{x} - \phi)dt + \int_0^1 N(S)dt + N(\psi)_1 \qquad (21)$$

and the error in the optimality conditions,

$$Q = \int_0^1 N(\dot{\lambda} - f_x + \phi_x \lambda)dt$$
$$+ \int_0^1 N(f_u - \phi_u \lambda + S_u \rho)dt$$
$$+ N[\int_0^1 (f_\tau - \phi_\tau \lambda + S_\tau \rho)dt + (\psi_\tau \mu)_1]$$
$$+ N(\lambda + \psi_x \mu)_1 \qquad (22)$$

are defined, where $N(\nu)$ denotes the squared norm of a vector ν, i.e.

$$N(\nu) = \nu^T \nu. \qquad (23)$$

For the exact, optimal solution,

$$P = 0, \qquad Q = 0. \qquad (24)$$

However, as approximation to the optimal solution, the numerical method aims at

$$P < \epsilon_1, \qquad Q < \epsilon_2, \qquad (25)$$

where ϵ_1 and ϵ_2 are small, prescribed numbers. More details about the CGS technique are described in, for example, (Wu and Miele, 1980) and (Miele and Iyer, 1970).

The calculations described below will be implemented under the convergent conditions of $\epsilon_1 < 0.1^{-10}$ and $\epsilon_2 < 0.1^{-4}$.

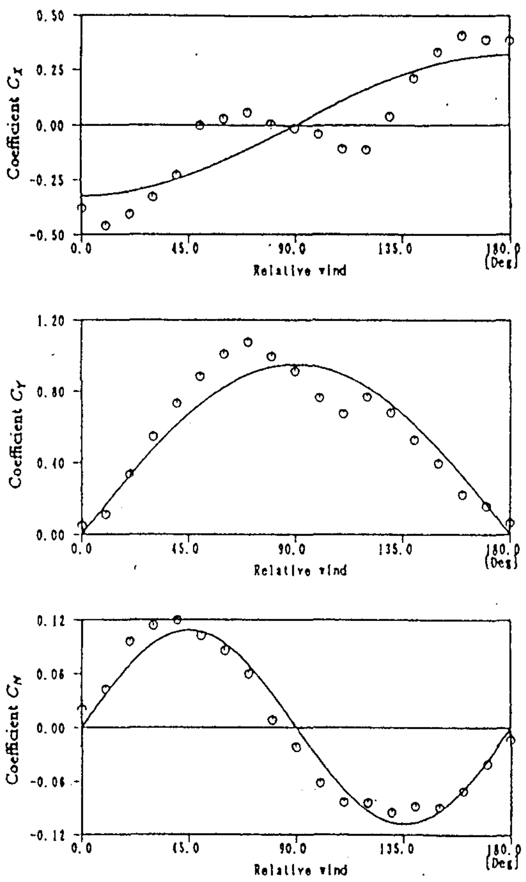

Fig.2 Wind Effects,C_X, C_Y and C_N

5. MINIMUM-TIME DEVIATIONS WITH WIND DISTURBANCES

5.1. *Wind Effects on the Ship*

Figure 2 shows the effects of wind pressures on the ship in the fore-and-aft and athwart directions, and its turning moment in the *Shioji Maru*. The small circles in each figure denote the empirical results, and the solid lines, the first approximations of them in eq. (13)-eq. (15). As can be seen from these, the bow falls off the wind, when the ship encounters the wind forward of the beam, while it turns away from the wind, when it encounters the wind aft of the beam. These characteristics of the ship's behaviour are important effects that a ship's master must take proper account of in ship-handling with wind disturbances.

5.2. *The Problems and The Optimal Solutions*

The first examples are minimum-time deviation problems with wind disturbances. The ship must deviate 500 m away from the initial approach line in a minimum maneuvering time, using the rudder. The winds blow from the starboard bow and the port stern quaters at relative wind velocities of 20 m/sec and 30m/sec, respectively. The ship's initial approach speed is 12 knots, and the sideways speed must disapear and the head must be redirected on the original course after ending the deviation.

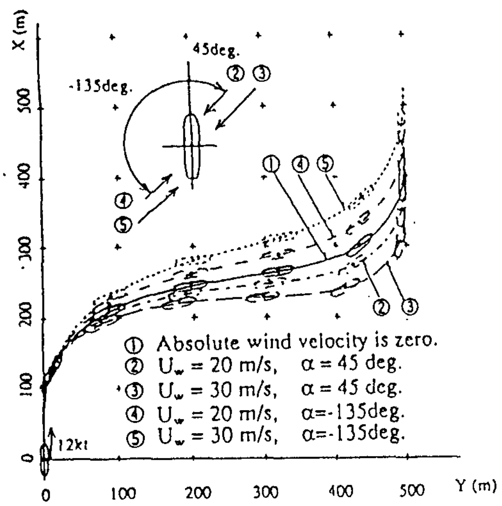

Fig.3 The Calculated Paths of Minimum Deviations with Winds

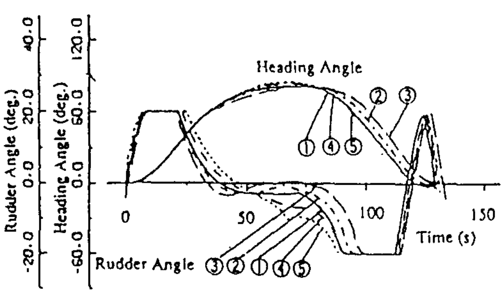

Fig.4 The Calculated Time Histories of Rudder Angles and Heading Angles

Figure 3 shows the optimal paths and ship's headings in each case, solved by the CGR method. Figure 4 shows the corresponding time histories of the rudder angles and the heading angles. It should be noted that the time histories of the heading angles have almost the same patterns, whereas those of the rudder angles are different in each case. Thus, it is generally concluded to be sufficient that a ship's master should pay attention only to maintaining the ship's heading along the minimum time solution with no wind, in order to accomplish the minimum-time deviation maneuvering.

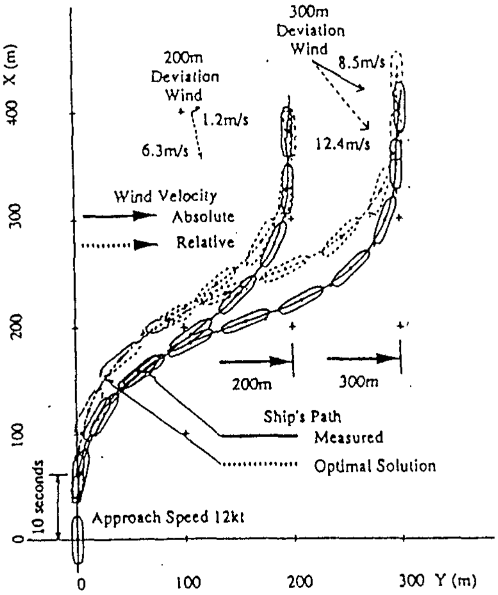

Fig.5 The Paths of the Actual Automatic Deviation Tests

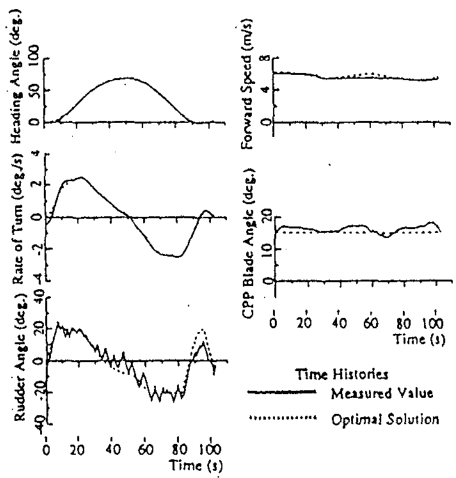

Fig.6 The Time Histories of Heading, Rates of Turn, Rudder Angles, Speeds and CPP Angles

5.3. *The Actual Sea Test*

In order to evaluate the reliability on the formulations and calculations, the following actual deviation tests were carried out at sea, using the *Shioji Maru*. In these trials, the distances between the first approach course and the final one were set up as distances of 200m and 300m. The control law implementing the calculated steering order was constituted in the following simple form:

$$\delta^*(t) = \delta_0(t) + K_1(\Psi_0(t) - \Psi(t)) + K_2(r_0(t) - r(t)), \qquad (26)$$

where $\delta_0(t)$, $\Psi_0(t)$ and $r_0(t)$ denote the optimal solutions of the steering order, the heading angle and the rate of turn.

Figure 5 shows the ship's actual paths, measured through a Doppler type of speed log (solid line), and the calculated ones (dotted line).The latter were calculated under conditions with no wind. It is of interest that although the intermediate paths differ slightly between the calculations and the actual tests, there are no large differences in the final positions in each case. Figure 6 shows the comparisons between the measured and calculated heading angles, rates of turn, the rudder angles and the forward speeds. The last two figures clearly demonstrate that the actual rudder angles and CPP blade angles faithfully follow the signals of the steering and pitch angles of the CPP ordered by the computer, and thus the actual heading angles and forward speed also coincide with the intended ones.

6. MINIMUM TIME INWARD STOPPING PROBLEMS WITH WIND DISTURBANCES

6.1. *Setting up the Problems*

As a second example of minimum-time maneuvering with wind disturbances, minimum-time stopping problems are considered.

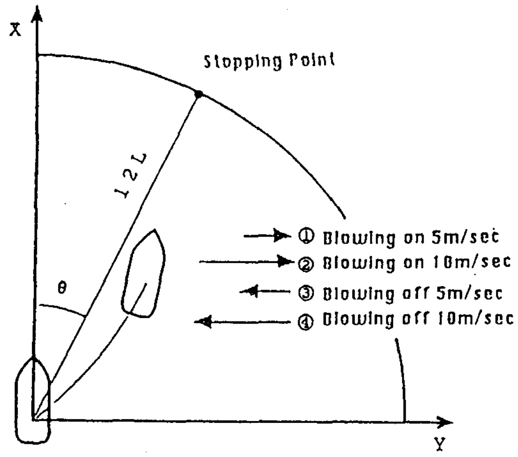

Fig.7 Minimum Stopping Problem with Wind

The problems treated here are set up as follows:

1. At the initial time, the ship is traveling at a position located 12 times her length (600m) from the final stopping point, whose bearing from her head is α degrees to the starboard side (Figure 7). Her speed is the normal sailing speed, namely 12 knots.
2. The winds blow on or blow off the final stopping point with a relative wind velocity of 5 or 10 m/sec.
3. The ship's head at the final stopping point must be redirected to the original course, in the attitude of the so-called "inward stopping".

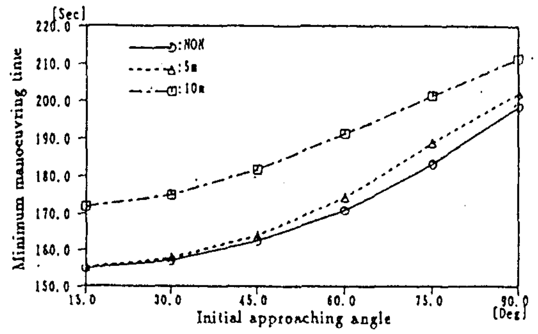

Fig.8 The Minimum Inward Stopping Times with the Wind Blowing On

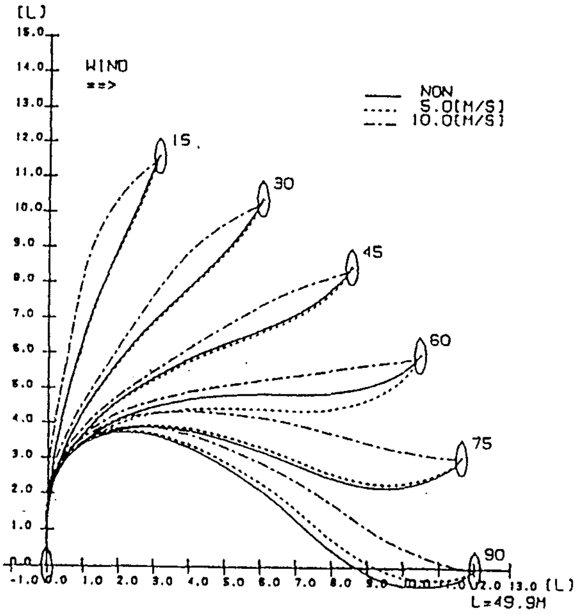

Fig.9 Paths of The Minimum-time Inward Stopping with Wind Blowing on

6.2. *Minimum Time Inward Stopping with Wind Blowing on*

Figure 8 shows the maneuvering time until stopping at the given point in the minimum inward stopping with the wind blowing on. It is noticed that the time differences between the minimum stopping with wind velocity 5 m/sec and with wind of 10 m/sec are longer than those between the minimum stopping with no wind and that with wind of 5 m/sec.

In order to understand the reason for this, the paths and other related maneuvering elements should be examined in detail.

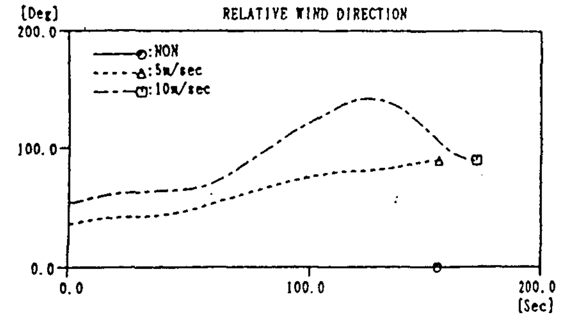

Fig.10 The Relative Wind Directions

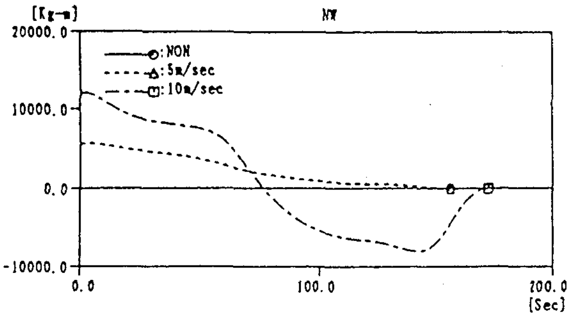

Fig.11 The Yaw Moments

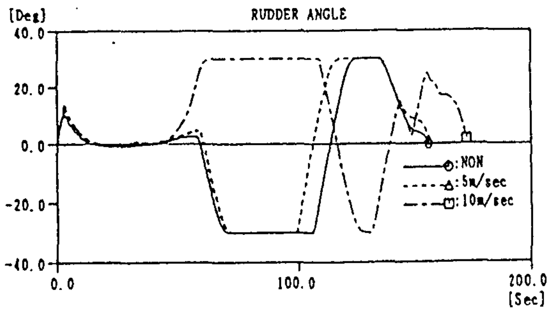

Fig.12 The Time Histories of Rudder Angles

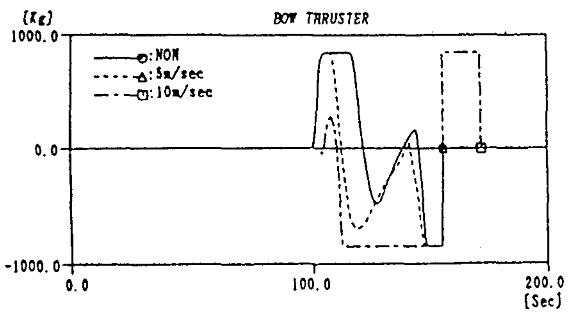

Fig.13 The Time Histories of Bow Thruster Forces

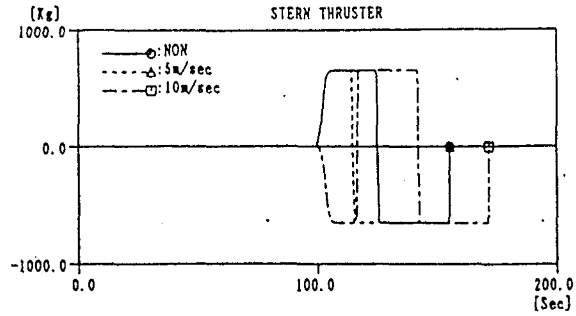

Fig.14 The Time Histories of Stern Thruster Forces

Figure 9 shows the ship's paths. From this figure, it is found that the paths when the wind velocity is 10 m/sec, swell out in the outside directions. Figures 10 and 11 show the time histories of the wind directions relative to her head, and the yaw moments when the initial bearing to the final point is 15 degrees. It is clearly recognized that when the wind velocity is 10 m/sec, the relative wind to her head changes to aftward from the abeam direction after 100 sec, due to her swelling out path.

As the result, the yaw moment in the last stage, approaching the final point, changes from a starboard moment to a port one. It is clear that the minimum inward stopping maneuvers with a wind speed of 10 m/sec take account of this wind effect. Figures 12, 13 and 14 are the time histories of the rudder angles, and the bow and stern thruster forces, respectively. It is noticed that the ship is turned under full power at the last stage just before the terminal, utilizing starboard steering, and the starboard bow and port stern thrusting , in addition to the wind effect described above.

6.3. *Minimum Time Inward Stopping with Wind Blowing off*

As a final example, the minimum-time inward stopping problem with wind blowing off the final point is discussed.

Figure 15 shows the maneuvering time before stopping at the final point in minimum time. Also, Figure 16 shows the paths taken. It is noted that differing from the last problem, all the inward-stopping maneuvers with wind blowing off the final point almost coincide with the paths with no wind disturbances. The reason for these coincidences is that a yawing moment to the port side is gained naturally, at the last approaching stage, due to the wind effects described above.

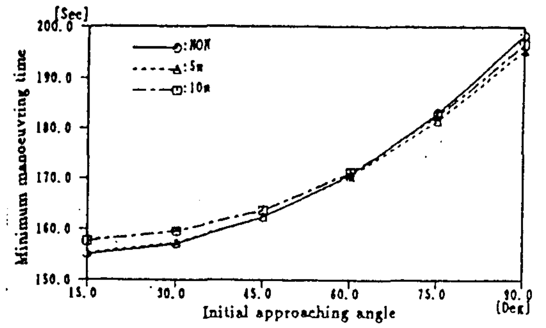

Fig.15 The Minimum Inward Stopping Times with the Wind Blowing Off

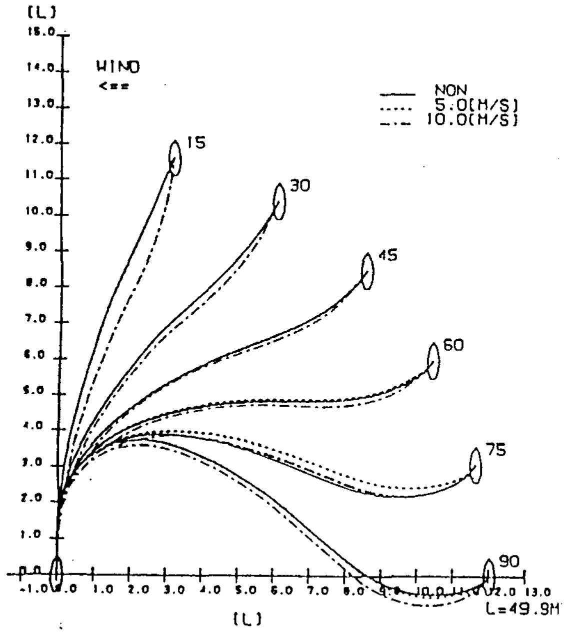

Fig.16 Paths of the Minimum-Time Inward Stopping with Wind Blowing Off

7. CONCLUSIONS

This paper has given the minimum-time maneuvering methods in two kinds of typical shiphandling problems under conditions with wind disturbances, for a small training ship with a rudder, a controllable pitch propeller, and bow and stern thrusters.

In the minimum-time deviation problem, it was concluded to be sufficient for a ship's master to pay attention only to maintaining the ship's course along the minimum solution with no wind, irrespective of how the winds are blowing. In this problem, furthermore, actual sea trials were implemented. As an interesting result, it was found that despite the intermediate paths in the tests being slightly different from the optimal solutions, there are no large differences at the final point in each case. In the problem of minimum-time inward stopping at a given position with the winds blowing on and blowing off, it was confirmed that from the viewpoint of shiphandling practice, the optimal solutions are reasonable maneuvering methods that make maximum use of the effects of the wind, especially in the minimum inward stopping problems with the winds blowing on.

8. ACKNOWLEDGEMENTS

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