TUMSAT-OACIS Repository - Tokyo

University of Marine Science and Technology

(東京海洋大学)

Structure of Uncertainties in Robust Nonlinear Control

メタデータ	言語: eng 出版者: 公開日: 2008-03-19 キーワード (Ja): キーワード (En): 作成者: 章, 菲菲 , 陶山, 貢市 メールアドレス: 所属:
URL	https://oacis.repo.nii.ac.jp/records/1203

Structure of Uncertainties in Robust Nonlinear Control

Feifei Zhang , Koichi Suyama

1 Introduction

As an effective design approach to nonlinear systems, the feedback linearization method has attracted a great deal of interest. A lot of theoretical results have been made out. In practical application, however, it seems there exist some limitations caused by the approach itself. Among them, a remarkable defect is that the linearization depends entirely on a model of a given nonlinear plant, which makes the resulting control system highly sensitive to parameter uncertainties. Although there are several research reports in which a linearized system is compensated by a robust control, such as H-infinity control, the robustness against parameter uncertainties in linearized system does not necessarily correspond to that in nonlinear ones.

Here, from a viewpoint of robust control design by the feedback linearization approach, it is ideal that the feedback linearization must be hold by perturbated parameters as well as nominal ones, then the robustness against parameter uncertainties can be guaranteed by applying linear robust control to the linearized model.

Of course, not all kind of parameter uncertainties can be treated in such a way. But, it is very important to clearify the structure of above-mentioned nonlinear model uncertainties. To such class of nonlinear systems, the physical meaning of parameter uncertainties are preserved in the linearizing process, therefore, the conservative design can be avoided. The another advantage of this approach exclude the state-dependence of model uncertainty which occurs usually in the case where parameter uncertainties are drawn together to one model uncertainty.

The purpose of this paper is to propose a new concept on parameter uncertainties which can be entirely seperated from the linearization and be treated by the robust control in section 2. To a class of nonlinear systems which is widely used, the structure of such parameter uncertainties is derived out in section 3. A practical example is presented to examine the result in section 4.

2 Problem Statement

Before stating our problem, let's review the feedback linearization approach. Consider a nonlinear single-input single-output system desicribed by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u \tag{2.1}$$

where $\boldsymbol{x}(t) \in R^n$ is a state vector, $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are real smooth vector fields on R^n, and u is a control input. The feedback linearization aims at finding a state transformation $\boldsymbol{z} = T(\boldsymbol{x})$ and a nonlinear feedback $u = \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x})v$ such that the system (2.1) can be rewritten by a linear state equation

$$\dot{\boldsymbol{z}} = A\boldsymbol{z} + Bv \tag{2.2}$$

in terms of a new state variable $\boldsymbol{z}$ and a new input v. The existence condition of such $T(\boldsymbol{x})$, $\alpha(\boldsymbol{x})$ and $\beta(\boldsymbol{x})$ is as follows.

1. the vector fields $\left[\boldsymbol{g}, ad_{\boldsymbol{f}}\boldsymbol{g}, \cdots, ad_{\boldsymbol{f}}^{n-1}\boldsymbol{g}\right]$ are linearly indenpendent.

2. $\left[ad_f^i \boldsymbol{g}, ad_f^j \boldsymbol{g}\right] \in span\left[\boldsymbol{g}, ad_f \boldsymbol{g}, \cdots, ad_f^{n-2}\boldsymbol{g}\right]$, for $i, j = 0, 1, \cdots, n-2$

where [,] indicates Lee-Bracket, and $ad_f^i \boldsymbol{g}$ is defined by

$$\begin{cases} ad_f^0 \boldsymbol{g}(\boldsymbol{x}) &= \boldsymbol{g}(\boldsymbol{x}) \\ ad_f^i \boldsymbol{g}(\boldsymbol{x}) &= \left[\boldsymbol{f}, ad_f^{i-1}\boldsymbol{g}(\boldsymbol{x})\right] \end{cases} . \tag{2.3}$$

If the conditions 1 and 2 are satisfied, we can take

$$\boldsymbol{z} = T(\boldsymbol{x}) = \left[L_f^0 \phi(\boldsymbol{x}), L_f^1 \phi(\boldsymbol{x}), \cdots, L_f^{n-1}\phi(\boldsymbol{x})\right]^T \tag{2.4}$$

where $L_f^i \phi$ is defined by

$$L_f \phi(\boldsymbol{x}) = \frac{\partial \phi}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}), L_f^i \phi(\boldsymbol{x}) = L_f(L_f^{i-1}\phi(\boldsymbol{x})) \tag{2.5}$$

and $\phi(\boldsymbol{x})$ is a scalar function satisfying the following conditions:

$$\begin{cases} \frac{\partial \phi(\boldsymbol{x})}{\partial \boldsymbol{x}} ad_f^j \boldsymbol{g}(\boldsymbol{x}) = 0, \quad i = 1, 2, \cdots, n-2 \\ \\ \frac{\partial \phi(\boldsymbol{x})}{\partial \boldsymbol{x}} ad_f^{n-1} \boldsymbol{g}(\boldsymbol{x}) \neq 0 \end{cases} . \tag{2.6}$$

In terms of the new state variable $\boldsymbol{z}$ and the new input v, the system (2.1) can be represented in the following linear equation :

$$\dot{\boldsymbol{z}} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots\cdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \boldsymbol{z} + \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 1 \end{pmatrix} v. \tag{2.7}$$

From the linearization process, it is clear that (2.7) is strictly available only if the parameters in (2.1) are all fixed. In practical cases, however, the paremeters of a nonlinear system are usually uncertain, then the linear model (2.2) corresponds only to the one case represented by the value of the parameters. So, it is necessary to treat the problem in the following form :

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \Delta) + \boldsymbol{g}(\boldsymbol{x}, \Delta)u \tag{2.8}$$

where Δ denotes parameter uncertainty.

Here, we are interested in whether or not the system (2.8) can be transformed into

$$\dot{\boldsymbol{z}} = A(\Delta)\boldsymbol{z} + B(\Delta)v \tag{2.9}$$

by a suitable feedback linearization. In other words, we want to find the structure of the parameter uncertainty Δ which is invariant with respect to the feedback linearization. Then, the control input v stabilizing (2.9) for all Δ guarantees the robustness of the original nonlinear system (2.8).

[Definition 1]

The parameter uncertainty Δ in the system (2.8) is said to have property IFL (Independent of Feedback Linearization) if there exists a feedback linearization $\boldsymbol{z} = T(\boldsymbol{x})$, $u = \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x})v$ such that the system (2.8) can be represented as (2.9).

In this paper, as the first step, we consider the following kind of popular nonlinear systems with parameter uncertainty which often appears in most physicla systems, for example, mechanical system or electrical systems.

$$\Sigma(\boldsymbol{x}, \Delta, u) : \begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \cdots & \\ \dot{x}_{n-1} &= f_{n-1}(\boldsymbol{x}, \Delta) \\ \dot{x}_n &= f_n(\boldsymbol{x}, \Delta) + g_n(\boldsymbol{x}, \Delta)u \end{cases} \tag{2.10}$$

In terms of the above definition, the problem considered in this paper can be stated as follows :

[Problem] Given a system $\Sigma(\boldsymbol{x}, \Delta, u)$, find the class of Δ which has IFL property.

3 Main Result

For a given system (2.10), it can be shown that the nominal system $\Sigma(\boldsymbol{x}, 0, u)$ is feedback linearizable. To the perturbated case, the system can be linearized into the form (2.9) by the identical feedback linearization if the parameter uncertainty has the following IFL property :

[Theorem]

The parameter uncertainties Δ in system (2.10) have IFL property if the elements f_{n-1} and f_n can be written by the following forms:

a. $f_{n-1}(x,\Delta) = a_1(\Delta)x_2 + a_2(\Delta)x_3 + \cdots + a_n(\Delta)f_{n-1}(x)$

b. $f_n(x,\Delta) = b_1(\Delta)x_2 + b_2(\Delta)x_3 + \cdots + b_n(\Delta)f_{n-1}(x)$

where, $a_i(\Delta)$, $b_i(\Delta)$, $i = 1,2,\cdots,n$ are functions of Δ.

(Brief proof)

Consider the following transformaton:

$$z = \begin{pmatrix} z_1 \\ \cdots \\ z_{n-1} \\ z_n \end{pmatrix} = \begin{pmatrix} x_1 \\ \cdots \\ x_{n-1} \\ f_{n-1}(x) \end{pmatrix}. \qquad (3.1)$$

For the nominal system $\Sigma(x,0,u)$, the last element z_n is equivalent to $\dot{x}_{n-1}$. Differentiating z yields

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \cdots \\ \dot{z}_{n-1} = z_n \\ \dot{z}_n = \frac{\partial f_{n-1}}{\partial x_1}x_2 + \cdots + \frac{\partial f_{n-1}}{\partial x_{n-2}}x_{n-1} \\ \qquad + \frac{\partial f_{n-1}}{\partial x_{n-1}}f_{n-1} + \frac{\partial f_{n-1}}{\partial x_n}(f_n + g_n u). \end{cases} \qquad (3.2)$$

Then, selecting a nonlinear input u as follows yields a linearized model of the form (2.9).

$$u = -\frac{1}{g\frac{\partial f_{n-1}}{\partial x_n}}\left(\frac{\partial f_{n-1}}{\partial x_1}x_2 + \cdots + \frac{\partial f_{n-1}}{\partial x_{n-2}}x_{n-1} + \frac{\partial f_{n-1}}{\partial x_{n-1}}f_{n-1} + \frac{\partial f_{n-1}}{\partial x_n}f_n - v\right). \qquad (3.3)$$

In the pertubated system case, however, the right side of $\dot{x}_{n-1}$(equation(2.10)) becomes $f_{n-1}(x,\Delta)$ which causes the form of $\dot{z}_{n-1}$ to be different from that in the nominal case. Also, the last term of u is changed. Using the relations a and b in the theorem, the following linear equation can be obtained which coincides with the form (2.9)

$$\dot{z} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots\cdots & & & & \\ a_1(\Delta) & a_2(\Delta) & a_3(\Delta) & \cdots & a_n(\Delta) \\ b_1(\Delta) & b_2(\Delta) & b_3(\Delta) & \cdots & b_n(\Delta) \end{pmatrix} z + \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 1 \end{pmatrix} v. \qquad (3.4)$$

4 Example

In this section, we present a practical design example to show how the parameter uncertainty in the nominal nonlinear plant is attenuated by a linear robust control method. The design effects are evaluated by simulation results.

The controlled object is a magnetic levitating system shown in figure 1, which consists of an electromagnet, an iron ball and a gap sensor. The iron ball is levitated by an attraction force f caused by the electromagnet which is controlled by electricity i.The control objective is to keep the ball at some position h_0 against the gravity.

$$L\dot{i} + Ri = u \qquad (4.1)$$

Suppose that the ball moves only in the vertical direction, then the dynamics of the plant can

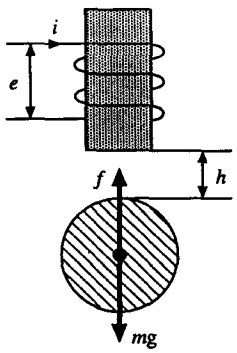


Fig.1 Magnetic Levitating System

be represented by

$$m\frac{d^2h}{dt^2} = \theta mg - f \tag{4.2}$$

$$f = K\frac{i^3}{(h+H)^2} \tag{4.3}$$

$$L\frac{di}{dt} + Ri = e \tag{4.4}$$

where, the parameter K and H are determined by the physical properties of the electromagnet and the iron ball. L and R denote the impedance and resistance of the coil. The parameter θ is an uncertainty factor used to represent changeable mass of the iron ball practical system. Let

$$\boldsymbol{x} = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T = \begin{pmatrix} h & \dot{h} & i \end{pmatrix}^T. \tag{4.5}$$

then, the nolinear state equatioon of the system is of the following form

$$\dot{\boldsymbol{x}} = \begin{pmatrix} x_2 \\ \frac{-Kx_3^2}{\theta m(H+x_1)^2} + g \\ -\frac{R}{L}x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix} u \tag{4.6}$$

It can be shown that a scalar funtion

$$\phi(\boldsymbol{x}) = x_1 \tag{4.7}$$

is suitable to construct a state transformation

$$\boldsymbol{\xi} = T(\boldsymbol{x}) = \begin{pmatrix} \phi(\boldsymbol{x}) \\ L_f^1\phi(\boldsymbol{x}) \\ L_f^2\phi(\boldsymbol{x}) \end{pmatrix} \tag{4.8}$$

such that

$$\dot{\boldsymbol{\xi}} = \begin{pmatrix} \xi_2 \\ \xi_3 \\ \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x})u \end{pmatrix} \tag{4.9}$$

where

$$\alpha(\boldsymbol{x}) = \frac{KRx_3}{mL(H+x_1)} + \frac{Kx_2x_3}{n(H+x_1)^2} \tag{4.10}$$

$$\beta(\boldsymbol{x}) = -\frac{K}{mL(H+x_1)} \tag{4.11}$$

Let the control input $u(t)$ be

$$u = -\frac{\alpha(\boldsymbol{x})}{\beta(\boldsymbol{x})} + \frac{v}{\beta(\boldsymbol{x})}, \tag{4.12}$$

then a linearized model of the plant can be obtained as follows:

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\theta} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \\ &=: A(\theta)\boldsymbol{\xi} + Bv \end{aligned} \tag{4.13}$$

The form of the above equation is nothing but (2.9).

To the linearized model (4.13), a robust control method for linear systems can be used to design the input v. The following is a test for the performance of the exact linearization method. Let

$$v = \begin{pmatrix} f_1 & f_2 & f_3 \end{pmatrix}^T \xi = f_1\xi_1 + f_2\xi_2 + f_3\xi_3 \tag{4.14}$$

be the state feedback such that the closed-loop system

$$\dot{\boldsymbol{\xi}} = (A(\theta) + BF)\boldsymbol{\xi} \tag{4.15}$$

is stable for all the θ. Then the resulting control

$$\begin{aligned} u &= Rx_3 + \frac{Lx_2x_3}{H+x_1} \\ &\quad - \frac{mL(H+x_1)}{2Kx_3}\left[f_1x_1 + f_2x_2 + f_3\frac{-Kx_3^2}{m(H+x_1)^2} + f_3g\right] \end{aligned} \tag{4.16}$$

robustly stabilizes the original system (4.6) against uncertainty θ. Figure 2 shows the responses $h(t)$ in case where mass-parameter changes when the controller (4.16) is used. The mass is changes as $m = 0.4kg$, $m = 0.52kg$, $m = 0.7kg$, while the nominal mass used to determine the controller is $0.52kg$. From the Figure 2 it is clear that all the curves show good responses .

5 Conclusion

In this paper, a new concept on parameter uncertainties in roubst nonlinear control has been proposed. To a class of nonlinear systems, the structure of such parameter uncertainties has been derived . A practical example has been presented to examine the result.

Reference

[1] A. Isidori, "Nonlinear Control Systems," Springer-Verlag, New-York, 1989.

[2] C.I. Byrbes and A. Isidori, " Steady State Response, Separation Principle and the Output Regulation of Nonlinear Systems," Proc. 28th CDC, pp.2247-2251, 1989.

[3] J.J. Slotine and W. Li, " Applied Nonlinear Control," Prentice-Hall, 1991.

[4] F. Zhang and K. Suyama, "H_∞ Control of Magnatic Suspention Systems," Proc. of the 3rd IEEE CCA, Vol.1, pp.605-610, 1994.

[5] F. Zhang and K. Suyama, " Nonlinear Feedback Control of Magnetic Levitating System by Exact Linearization Approach," Proc. 4th IEEE CCA, pp.267-268, 1995.

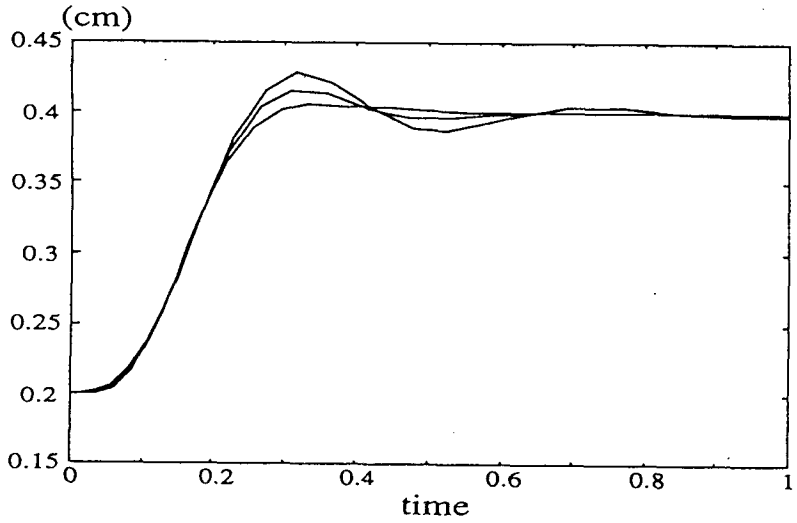


Fig.2 Step response of gap $h(t)$ with mass changes